MH370 BFO Models

Henrik Rydberg

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1 BFO model basics

In order to explain the recent models [1] of the Burst Frequency Offset, which accurately reproduces Inmarsat's original BFO graph [2], it is useful to start simple. The basic principle can be found in the V1 model [3], which does not take rate-of-climb into account. This model was first discussed on the TMF blog already in April 2014 [4].

Preliminaries

We will use a satellite-centered coordinate system here, because it will simplify analysis later on, when looking at the dependency between and along ping rings. We put the point of origin up in space, at an altitude of 35785300 meters above the imaginary stationary point (0N, 64.5E). We define the y axis as extending from that point towards the center of the earth. The x axis extends towards the east, and the z axis extends towards the north. This creates a right-handed coordinate system. Vectors are denoted in bold, and superscripts are used for both normalized and special scaled vectors. The length of a vector is denoted by the letter itself.

The Effect of Aircraft-Corrected Doppler

Let $\mathbf{p}$ be the position of the aircraft. Let $\mathbf{s}$ be the position of the satellite. Let L be the difference in distance between aircraft-to-satellite and aircraft-to-fixed-satellite,

$$L = |\mathbf{s} - \mathbf{p}| - p. \tag{1}$$

We define the doppler in the usual way, as the rate of change which is positive when objects move towards each other,

$$D = -F \frac{dL}{dt}. \tag{2}$$

Here, F is the transmission frequency divided by the speed of light. In our L-band case, $F = 1640/300$.

Let $\mathbf{v}_p$ be the velocity of the aircraft, and let $\mathbf{v}_s$ be the velocity of the satellite. Differentiating by parts be obtain

$$D = -F\left(\mathbf{v}_p \cdot \nabla_p L + \mathbf{v}_s \cdot \nabla_s L\right). \tag{3}$$

Since $s \ll p$, we can expand L around $\mathbf{s} = 0$. To second order (with typical values of $s/p < 0.03$), the expansion is already pretty good. We get

$$L^{(2)} = -\mathbf{s} \cdot \hat{\mathbf{p}} + s^2/2p. \tag{4}$$

Evaluating Eq.(3) we obtain

$$D^{(2)} = F\left(\hat{\mathbf{p}} \cdot \mathbf{v}_s + \bar{\mathbf{s}} \cdot (\mathbf{v}_p - \mathbf{v}_s) - \left(\bar{\mathbf{s}} \cdot \hat{\mathbf{p}} - \bar{s}^2/2\right) \hat{\mathbf{p}} \cdot \mathbf{v}_p\right). \tag{5}$$

Here, $\bar{\mathbf{s}}$ is a conveniently scaled position, $\bar{\mathbf{s}} = \mathbf{s}/p$. Since $\mathbf{s}$ is essentially perpendicular to $\mathbf{p}$, the last term is a very small correction. For clarity, we can thus construct the even simpler first-order approximation,

$$D^{(1)} = F\left(\hat{\mathbf{p}} \cdot \mathbf{v}_s + \bar{\mathbf{s}} \cdot (\mathbf{v}_p - \mathbf{v}_s)\right). \tag{6}$$

The first term is the satellite velocity projected onto the aircraft unit vector. The second term is the difference in aircraft and satellite speed, projected onto the scaled satellite vector. Since $\bar{s}$ is a small number, the satellite velocity dominates the expression for most aircraft speeds and headings.